MATHIN' AROUND THE HOUSE

Student/Class Goal
Students want to be able to find area and perimeter of any shape without the need to memorize a new formula each time.

Outcome *(lesson objective)*
Students will apply prior knowledge of perimeter and area by using irregular shapes. In addition, trigonometry will be introduced through the use of the Pythagorean theorem which will allow students to sketch appropriate diagrams for contextual problems and then solve these problems.

Time Frame

2 hours

Standard *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

NRS EFL
Levels 5-6

Number Sense	**Benchmarks**	**Geometry & Measurement**	**Benchmarks**	**Processes**	**Benchmarks**
Words to numbers connection		Geometric figures	6.6	Word problems	6.26
Calculation	5.4	Coordinate system		Problem solving strategies	5.26
Order of operations		Perimeter/area/volume formulas	**6.8**	Solutions analysis	5.27
Compare/order numbers		Graphing two-dimensional figures		Calculator	6.29
Estimation		Measurement relationships		Math terminology/symbols	5.29
Exponents/radical expressions	6.5	Pythagorean theorem	**6.11**	Logical progression	5.30
Algebra & Patterns	**Benchmarks**	Measurement applications	6.12	Contextual situations	6.32
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions	**6.16**	Rounding	6.14	Logical terms	5.33
Linear/nonlinear representations		**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	6.36
Linear equations		Data displays construction		Independence/range/fluency	5.36
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			

Vocabulary
Hypotenuse: In a right triangle, this is the side opposite the right angle.
Pythagorean Theorem: $a^2 + b^2 = c^2$

Materials
Calculators (Can be done without, but easier with)
SmartPal sleeves/eraser cloths/dry erase markers
Up on the Rooftop—Handout
Rulers
Irregular Perimeters and Areas—Handout
Yard Dimensions—Handout

Learner Prior Knowledge
Evaluating squares and square roots
Perimeter/Area of: Squares, Rectangles, Triangles, and Circles
Vocabulary related to right triangles and circles.

Instructional Activities
Step 1: Brief review of taking squares and square roots (with calculator). Also review the concepts of perimeter v. area and the formulas for rectangles, triangles, and circles. It may also be beneficial to refresh their vocabulary when it comes to right triangles. They should know the difference between the hypotenuse and the legs and that the hypotenuse is *always* the longest of the three sides.

Step 2: We want our students to be able to follow a sequence of steps when solving problems. Whether they know it or not, they probably already sort of do this. We want them to follow Polya's four step process:

1. Understand the problem (*What is the unknown? The data? The conditions?*)
2. Pick a strategy to solve the problem (*Have you seen a similar problem? One with a similar unknown?*)
3. Implement that strategy to come to a solution
4. Review the work and the solution to make sure the solution makes sense in the given context.

After step 4, if there seems to be an error with the solution, students should go back to step 1 and repeat the process until they come to a solution that makes sense.

For the first few lessons, these steps should be discussed and written down so that students can refer to them as a guide when solving problems. During the *I do* steps, your thinking aloud should show you going through all four steps in the process.

Step 3 : Pythagorean Theorem

- (*I do*) For this activity, students will need calculators, the Up on the Rooftop handout, and the SmartPal supplies. Read the first problem out loud, or have a student read it out loud. As the picture is not drawn to scale, we cannot use a ruler to find the correct height of the ladder. However, it should be noted that a triangle is formed by the house, ground, and ladder. Since the house is perpendicular to the ground, we actually have a right triangle, which has some special properties. Give them the equation for the Pythagorean Theorem. If you have technology capabilities, you can show them the proof as to why this works, or you can try to do it as a class. (This could also be something done later as it is a little algebra intensive.) Remember to "think aloud" as you go through the Polya steps.
    1. If you need to, reread the scenario for the problem. We are looking for the ladder height. Since the ladder will need to lean and still reach the top of the house, the ladder should be at least as tall as the house, or at least 18 feet.
    2. The strategy for any problem where you need to find the side of a right triangle is to use the Pythagorean Theorem. Many times, however, it is also useful to draw a picture in order to explicitly see which sides are legs and which one is the hypotenuse. In this case, the picture has been drawn for us, so we can move right into using the equation.
    3. Based on the picture, we see the ladder is the side opposite the right angle, or the hypotenuse. If the Pythagorean Theorem is given as $a^2 + b^2 = c^2$, then a and b are the legs and c is the hypotenuse. Thus, we will plug in the two known values for a and b and our unknown value h for c. This gives the equation: $7^2 + 18^2 = h^2$. Using our calculators, we find that this becomes: $49 + 324 = h^2$. Simplifying yields: $373 = h^2$, and once more using our calculator we get that h is about positive or negative 19.3 (the answer could also be left in radical form). As a length cannot be negative, we know our ladder must be at least 19.3 feet in length.
    4. To reflect on this, we look back to the problem. We wanted a ladder that would reach to the roof of an 18 foot high house if the feet of the ladder were placed 7 feet from the house. In step 1, we knew that this meant the ladder had to be at least as tall as the house. We found a ladder length over a foot more than the height of the house so our answer does make sense.
- (*We do*) The second problem on the handout is a similar situation with new numbers and the unknown in a new spot. Discuss with the class this problem, guiding them through the four step process.
    1. This time we are looking for the height of the house. Since the ladder leans and still reaches the top of the house, we know that the house must be shorter than the ladder. So the house must be shorter than 25 feet.
    2. Once again, the picture is already drawn for us with distances put in. As we have a right triangle problem where we want to find a side length, we will use the Pythagorean Theorem.
    3. Again, the ladder is the hypotenuse and will take the place of c in the equation. This time, however, that is not the unknown. On the left hand side of the equation, there are two variables, but they both represent the legs. It will not matter which leg goes in for which variable. In this situation, our unknown is one of the legs, so our equation becomes: $7^2 + h^2 = 25^2$. Order of operations tells us to do exponents first, so we have: $49 + h^2 = 625$. Now we want to get h alone, so we subtract 49 from both sides, giving: $h^2 = 576$, and then taking square roots, we get that h is positive or negative 24. Once again, lengths are positive, so we throw out the negative 24 and our answer is positive 24 feet.
    4. In step 1, we said our answer needed to be less than 25 feet as that was all the taller our ladder was. As we got an answer that meets those expectations, our solution makes sense.
- (*You do*) Have the students do the final problem on the handout. It will be similar to the one you did in the previous section. This time, however, they must fill in the appropriate lengths in the picture based on the word problem.

Step 4 : Perimeter

- (*I do*) Segue from finding single side lengths to finding distances around entire shapes. For this part, everyone will now need the handout on irregular areas and perimeters, the SmartPal supplies, and a ruler.
    1. Read the first scenario aloud to the class about the basketball court. Then read the first question (just the part about perimeter, not the area question as well). As part of the understanding of this problem, we must figure out exactly which parts of the diagram we must find lengths for. As this is an irregular shape, we must break it up into shapes we know. We must find the bottom edge, the two slant sides, and the semicircle at the top.
    2. As we need to find multiple lengths, we will need to incorporate a few different strategies. The drawing has some dashed lines already drawn in to help us break this into regular shapes that we can work with. We see that we'll have to take half of the circumference of the circle at the top, the length of the bottom is given to us, and we'll have to find the length of the hypotenuses of the right triangles on the sides. Our "strategy" will be to use the correct formulas for each part and then add the four lengths together to get our final perimeter.

3. Since C=πd and in the picture d=3.7meters, we know the circumference of the entire circle is 3.7π meters, or about 11.618 meters. To find just the top half then, we divide this by 2 to get that the circumference of the top semicircle is 1.85π meters or about 5.809 meters. (Note: It is personal preference whether you have them leave their answers in terms of π[recommended] or use a calculator to find rounded answers.) It should be noted that the two right triangles on the side are congruent, so we must only use the Pythagorean Theorem once. Doing so as above, we find that each slanted side of our shape is about 5.91 meters. Now we just have to add all of our sides: 5.809 meters (semicircle) + 5.91 meters (one slant side) + 5.91 meters (the other slant side) + 6 meters (the bottom) = 23.629 meters for the entire perimeter.
4. As a reflection, we know first that we're finding a distance so we should have a positive number. We do. We also know that it should be greater than any of the four parts we added together without being wildly large compared to our values (since each individual length was about 6 meters, we knew we should get something close to 6*4=24 meters). We also have this condition met. Therefore our answer definitely seems reasonable.

- (*We do*) The second scenario contains the same idea. We have an irregular shape, this time composed of semicircles and rectangles. This time, get the class as a whole to discuss how to solve the problem.
  1. After getting someone to read the scenario and just the first question about distance around the outside edge, the understanding step comes in. Here, students should think about that "outside edge" part of the question. What exactly do we need to find? The bigger semicircle circumference and the very top and bottom straight edges.
  2. We are told that the curves on the end are actually semicircles. So the students should fill in the rest of the circle so they can see the full shape they will be finding the circumference of. Then we will use the same strategy as with the basketball paint area. We will use the appropriate formulas to find perimeters of the different parts and then add our lengths together.
  3. We're told the straightaways are 25 feet and we see there are two of those. That just leaves the curved portions. The two semicircles share a center, and we are told the smaller circle has a radius of 10 feet. The bigger circle has an additional 5 feet for its radius, so the radius of the larger circle is 15 feet. Since C=πd, and d=2r, we have C=2πr=2*π*15=30π feet or about 94.2 feet. Technically, we have a semicircle, so we should next divide this by 2 to give us 47.1 feet. Now we have to add everything together: 25 feet (one straightaway) + 25 feet (2nd straightaway) + 47.1 feet (one curve) + 47.1 (2nd curve) = 144.2 feet around the outside edge of the track.
  4. Our reflection is similar to above. As a distance, we need a positive value. We also need it to be bigger than the parts that we found. (Another estimation we could have done was the 50 ft for the 2 straightaways, and then chop off the curves and use just the diameter of the larger circle which we know to be 30 ft on each side. This would have given us 50+30+30=110ft as an estimate and we would know our actual answer would be larger since the curve takes longer to go around than the straight diameter would.)
- (*You do*) Have the students do the final perimeter problem on the handout (just the first question of the last scenario). It will be similar to the one you did in the previous section. This time, however, they must fill in the appropriate lengths in the picture based on the word problem. They need to make sure to not count the entire 44 yards at the top since they will skip a portion of the straight edge to go around the semicircle.

Step 5: Area

- (*I do*) Using the same materials, you will go back and do the 2nd problem of each scenario. Now we will be dealing with the *interior* of each shape instead of the exterior. (Remember to think aloud as you solve the problem!)
  1. Reread the scenario aloud to the class as well as the question on the area. To understand the problem, students need to recognize that we want the area of the *entire* interior, not just the trapezoid at the bottom.
  2. Just like with perimeter of an irregular shape, we need to break it up into shapes that we have area formulas for. This picture has it done for us, so we have to take the two right triangles at the bottom, the rectangle at the bottom, and the semicircle at the top. Make sure to mention that using the entire circle at the top gives you another irregular shape at the bottom that would be harder to split into known shapes.
  3. While doing perimeter we noticed that the two right triangles were congruent. This means we need only find the area of one as the area of the other will be the same. To find the area, we take the base (1.15m) times the height (5.8m) to get 6.67m^2. For the rectangle, we take the length (5.8m) times the width (3.7m, which can be found from the foul line at the top or by subtracting the two 1.15 meter sections from the entire 6 meter bottom) to get 21.46 m^2. Finally, the semicircle at the top is one-half of the circle with $A=\pi r^2= \pi*(3.7/2)^2 = \pi*(1.85)^2 = 3.4225\pi$ m^2 or about 10.75 m^2 for the entire circle. The semi-circle, then, would be about 5.375 m^2. Adding these together we get: 6.67 m^2 + 21.46 m^2 + 5.375 m^2 = 33.495 m^2.
  4. We need to check that we have a positive value as area is always positive. We do. We also need to make sure we put the correct units. Units for area are always squared, which we have.
- (*We do*) The second scenario contains the same idea. We have an irregular shape, this time composed of semicircles and rectangles. This time, get the class as a whole to discuss how to solve the problem.
  1. Have someone reread the scenario as well as the second question. This time, we want to find the area between the

edgelines.

2. There are two possibilities here.
   A)We can consider the entire space outlined by the outer edge. In that case, we'll find the area of the semicircles at the top and bottom and the rectangle formed using the two outer straightaways and the diameters of the semicircles. From that, we would subtract the interior created by the inside edges. That would be found using the two smaller semicircles and the rectangle created by the two inside straightaways and the diameters of the smaller circles.
   B)Our other option is to just find what is between the two sets of edges. We would then make the straightaways into rectangles. We would find the curved portion the same way as above, finding the area of the outside semicircle and subtracting the inside semicircle. We would then add up our parts to find the entire area.
3. Method A) The rectangle has dimensions 25ftx30ft, so the area is 750ft^2. The circle has area 225π ft^2 or about 706.5 ft^2. Instead of dividing by two as we have a semicircle, we should recognize that we have two congruent semicircles and we would just multiply by two later. So our area inside the outside edges is 750+706.5=1456.5 ft^2. The interior rectangle has dimensions 25ftx20ft, so the area is 500ft^2. The smaller circle has area 100π ft^2 or about 314 ft^2. Again, we have two congruent semicircles, so we will not divide by two. The interior area then is 500+314=814 ft^2. Subtracting the inside area, we get 1456.5-814=642.5 ft^2.
   Method B) If you choose this option, the two straightaways each have dimensions of 25ft x 5 ft, which gives them each an area of 125 ft^2. Looking at one of the curves, we have a circle with a radius of 10ft that we must subtract from a circle with a radius of 15 ft. Make sure to discuss that we do not need to worry about dividing by 2 for a semicircle at the end as we would just end up doubling our result because we have the same situation at the other end of the track. We end up with the area of the track curve = Area of the bigger circle – Area of the smaller circle = 225π – 100π = 125π ft^2 which is about 392.5 ft^2. Adding this to the two straightaways, each 125 ft^2 in area gives us 392.5+125+125=642.5ft^2.
4. We need to check that we have a positive value as area is always positive. We do. We also need to make sure we put the correct units. Units for area are always squared, which we have.

- (*You do*) Have the students do the final perimeter problem on the handout. It will be similar to the one you did in the previous section. This time, however, they must fill in the appropriate lengths in the picture based on the word problem. They need to make sure to not count the entire 44 yards at the top since they will skip a portion of the straight edge to go around the semicircle.

Assessment/Evidence (*based on outcome)*

Each of the *you do* steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the *we do* steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student's mastery of the concepts.

Give them the Yard Dimensions handout to use as an end of class (or homework) assignment. This covers perimeter and area of irregular shapes. When calculating the perimeter they will have to find the hypotenuse of a right triangle, thus it covers all topics taught in this lesson. This will allow instructors to assess whether or not students have a handle on the topics covered.

Teacher Reflection/Lesson Evaluation

Not yet completed.

Next Steps

Prove the Pythagorean Theorem.
Special right triangles (45-45-90, 30-60-90).
Areas and perimeters of more complicated shapes and volumes of irregular solids.

Technology Integration

http://www.mathsisfun.com/geometry/pythagorean-theorem-proof.html

Proof of the Pythagorean Theorem. There are also multiple sites online with interactive proofs of the Pythagorean Theorem.

Purposeful/Transparent

Students will need to be able to find the perimeter and area of any shape regardless on its regularity and they would like to make it as simple as possible, which means getting rid of the need to memorize multiple formulas. Teachers will show students how to break irregular shapes into shapes they can easily identify (triangle, rectangle, and circle), and then guide them in finding perimeters and areas of these irregular shapes. This way, students need only know three formulas.

Contextual

Geometry has many hands-on, real-world applications. Interior designers, carpenters, and architects must all worry about the perimeter and area of rooms in order to create accurate designs and materials. The Pythagorean Theorem is the basis for the distance formula. It is therefore useful in the airplane industry (making sure planes are a safe distance apart), finding optimal routes, and, once again, in the building fields.